

New York University  
Department of Physics

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

Fall, 2001

# Dynamics

1. Answer all problems.
2. If not otherwise indicated, all parts of a problem are equally weighted.
3. Write your own identification number on your answer booklets.

# Problem 1

Consider two particles making an elastic collision. The collision acts for a very short time, during which the interacting bodies exert very large transient forces on one another. Denoting the impulse of one of these forces as

$$\mathbf{S} = \int \mathbf{F} dt$$

which remains finite, show that Lagrange's equation for the  $j$ th degree of freedom becomes

$$\left(\frac{\partial L}{\partial \dot{q}_j}\right)_f - \left(\frac{\partial L}{\partial \dot{q}_j}\right)_i = S_j \quad ,$$

where the subscripts  $i$  and  $f$  refer to the state of the system before and after the impulse, and  $S_j$  is the impulse of the impulsive generalized force corresponding to coordinate  $q_j$ , which does not change appreciably during the collision. The Lagrangian includes all non-impulsive forces.

# Problem 2

Consider a planetary orbit whose eccentricity  $e$  is small ( $\ll 1$ ). Position vectors and polar angles relative to both the Sun and the empty focus of the elliptic orbit are shown in the figure below. Show that

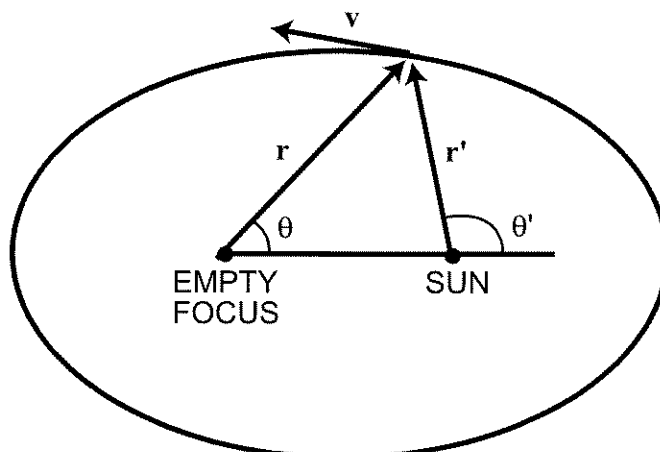
(a) the radial velocities  $dr'/dt'$  and  $dr/dt$  are both first order in  $e$ ,

(b) the angular velocities  $\dot{\theta}' = d\theta'/dt'$  and  $\dot{\theta} = d\theta/dt$  satisfy

$$r' \dot{\theta}' = v + O(e^2), \quad r \dot{\theta} = v + O(e^2),$$

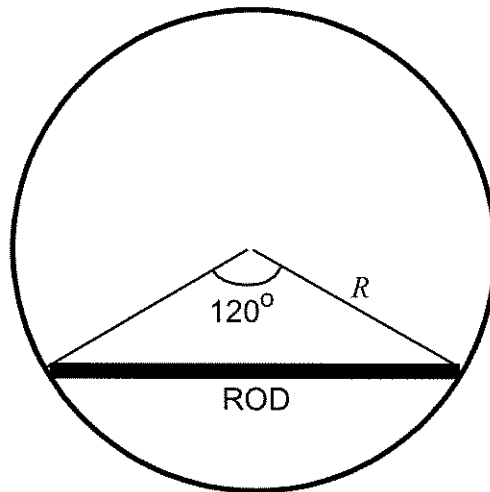
where  $v$  is the magnitude of the velocity,

(c) the Sun-centered angular velocity  $\dot{\theta}'$  is constant with an error **first** order in  $e$ , but the angular velocity  $\dot{\theta}$  relative to the empty focus is constant with a correction **second** order in  $e$ .



## Problem 3

In the diagram, the uniform rod of mass  $m$  moves with its ends in contact (no friction) with the vertical circle of radius  $R$ . Find its period for small oscillations, and find the length of a simple pendulum that has the same period.



## Problem 4

(a) Show that the transformation

$$Q = \arctan\left(\frac{\alpha q}{p}\right) \quad , \quad P = \frac{\alpha q^2}{2} \left( 1 + \frac{p^2}{\alpha^2 q^2} \right)$$

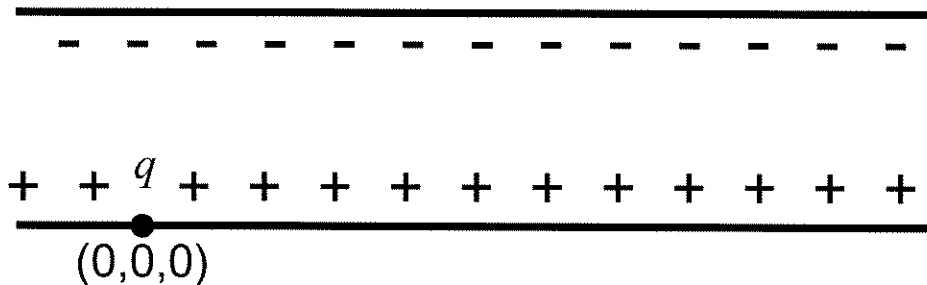
is canonical. As usual,  $q, p$  ( $Q, P$ ) are the old (new) coordinate and momentum in a system with one degree of freedom. Take  $\alpha$  to be constant.

(b) A particle moves on the  $x$ -axis under the influence of a one-dimensional potential  $V = C|x|^{1/2}$ ,  $C$  a positive constant. For periodic motion with energy  $E$ , find the action and use it to calculate the period.

## Problem 5

The figure shows a positive point charge  $q$  born at rest at the origin of a coordinate system located on the anode of a parallel plate capacitor. The capacitor is located in a uniform magnetic field directed out of the page. Letting  $\mathbf{E}$  and  $\mathbf{B}$  denote the electric and magnetic fields,

- write down the Hamiltonian of the system,
- use the Hamilton equations of motion to find the relevant time derivatives,
- determine the velocity and position as functions of the time.



## Problem 6

- A spring moves parallel to the  $x$ -axis. Assuming that the density is uniform and that the velocity of every point on the spring is linear in the undeflected distance from one of its ends, show that the kinetic energy of the spring is given by

$$T = \frac{M}{6} ( v_a^2 + v_b^2 + v_a \cdot v_b ) ,$$

where  $M$  is the mass of the spring, and  $v_a$  and  $v_b$  are the velocities of the endpoints  $a$  and  $b$ .

- The springs in the figure below have the same mass  $M$  and stiffness constant  $k$ . Using the results of part (a) to model the effect of the mass of the springs, find the eigenfrequencies of the system shown. Each of the particles has mass  $m$ , not equal to  $M$ .

