New York University
Department of Physics

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

Fall 2001

Electromagnetism, Part I

1. Answer all problems.
2. If not otherwise indicated, all parts of a problem are equally weighted.
3. Write your own identification number on your answer booklets.

Problem 1

A spherical cavity with permittivity ε_0 and radius R is introduced in an infinite uniform medium with permittivity ε and a previously uniform polarization $\mathbf{P}_0$. Find the electric field $\mathbf{E}$ inside and outside the cavity, assuming that the medium is a normal dielectric, with a linear relation between $\mathbf{P}$ and $\mathbf{E}$.

Problem 2

An infinitely long cylinder of radius R is filled with a dielectric medium with a permanent polarization $\mathbf{P} = a\rho\hat{\mathbf{e}}_\rho$ where ρ is the perpendicular distance to the cylinder's axis and a is a positive constant. The cylinder rotates around its axis with angular velocity ω ($\omega R << c$, so that retardation and relativistic effects can be neglected).

(a) Find the electric field $\mathbf{E}$ inside and outside the cylinder.

(b) Find the magnetic induction $\mathbf{B}$ inside and outside the cylinder, assuming $\mu = \mu_0$ everywhere.

(c) Find the electromagnetic energy per unit length before the cylinder starts rotating, and while it rotates with angular velocity ω, and explain the origin of the difference.

Use the SI system of units.

Note: you may find useful the following formula for cylindrical coordinates,

$$(\nabla \times \mathbf{B})_\phi = \frac{B_\rho}{z} - \frac{B_z}{\rho}$$

Problem 3

A spherical shell of radius a and charge Q, uniformly distributed over its surface, rotates counterclockwise about a diameter with angular velocity $\omega = \omega_0 + kt$ (ω_0 and k are constants). A small loop of radius $b << a$ and resistance R lies on the plane perpendicular to the rotation axis and its center coincides with that of the spherical shell.

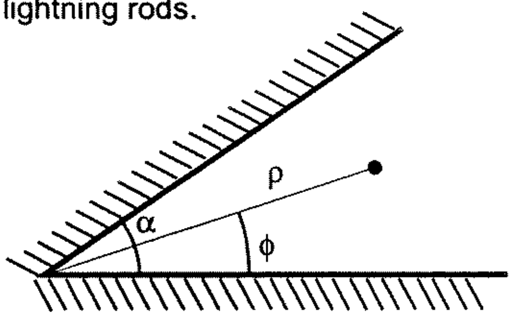
(a) Find the magnetic induction $\mathbf{B}$ at the center of the sphere.

(b) Find the current I induced in the small loop. Evaluate I numerically for Q = 100 C, a = 1 m, $b = 10^{-2}$ m, $k = 10^3$ sec^{-2}, $R = 10^{-2}\ \Omega$. (Recall $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}). Use the SI system of units.

Problem 4

(Do either (a) or (b), not both)

(a) Two conducting planes, held at potential V, intersect at an angle α. Derive the leading functional dependence of the potential Φ, the electric field $\mathbf{E}$, and the surface charge σ on the distance ρ to the corner and the azimuthal angle ϕ, for small values of ρ.
How does σ depend on ρ for $\alpha = \pi/2,\ 3\pi/2,\ 2\pi$? Use your result to explain qualitatively the effectiveness of lightning rods.



(b) A particle of charge e and mass m moves with velocity v in a direction parallel and at a distance b from the z-axis. A hypothetical magnetic monopole is at rest at the origin of coordinates, producing a magnetic induction

$$\mathbf{B} = \frac{g\mathbf{r}}{4\pi r^3} \quad .$$

(i) Neglecting, as a zeroth order approximation, the deflection of the charged particle, calculate the change in its angular momentum from $t = -\infty$ to $t = +\infty$.

(ii) Using Bohr's quantization rule for changes in angular momenta, derive the Dirac quantization condfition. Explain, without calculation, what happens to the conservation of angular momentum.

Note: you may find useful the formula, in cylindrical coordinates,

$$\nabla^2\Phi = \left[\frac{1}{\rho}\frac{}{\rho}\left(\rho\frac{}{\rho}\right)+\frac{1}{\rho^2}\frac{^2}{\phi^2}+\frac{^2}{z^2}\right]\Phi \quad ,$$

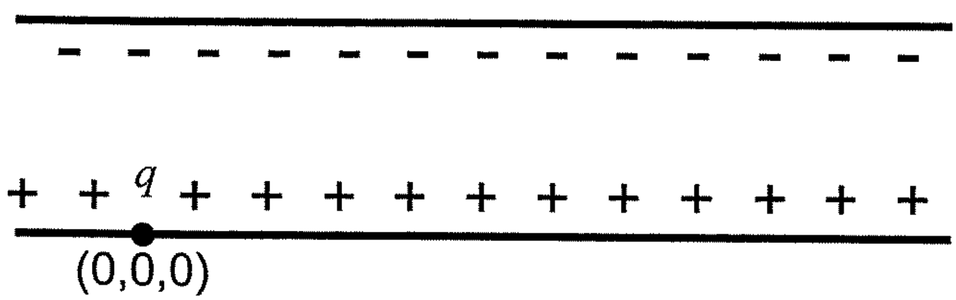
and the integral

$$\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^{\frac{3}{2}}} = 2 \quad .$$

Problem 5

The figure shows a positive point charge q born at rest at the origin of a coordinate system located on the anode of a parallel plate capacitor. The capacitor is located in a uniform magnetic field directed out of the page. Letting $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic fields,

(a) write down the Hamiltonian of the system,
(b) use the Hamilton equations of motion to find the relevant time derivatives,
(c) determine the velocity and position as functions of the time.



Problem 6

(a) A spring moves parallel to the x-axis. Assuming that the density is uniform and that the velocity of every point on the spring is linear in the undeflected distance from one of its ends, show that the kinetic energy of the spring is given by

$$T = \frac{M}{6} \left(v_a^2 + v_b^2 + \mathbf{v}_a \cdot \mathbf{v}_b \right) ,$$

where M is the mass of the spring, and $\mathbf{v}_a$ and $\mathbf{v}_b$ are the velocities of the endpoints a and b.

(b) The springs in the figure below have the same mass M and stiffness constant k . Using the results of part (a) to model the effect of the mass of the springs, find the eigen-frequencies of the system shown. Each of the particles has mass m, not equal to M.