

New York University

Physics Department

**PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE**

**QUANTUM MECHANICS I**

Fall, 2001

READ INSTRUCTIONS CAREFULLY

1. ANSWER ALL PROBLEMS.
2. All problems have the same point value. When a problem is divided into parts, their relative weights are given as percentages in parentheses. If not otherwise indicated, all parts are equally weighted.
3. Write your own identification number on your answer booklets.
4. Show ALL your work.

**Problem 1**

- a) Find the minimum pressure exerted by a particle on the walls of a one-dimensional infinite well of width  $L$ .
- b) Find the average electrostatic potential due to a hydrogen atom in its ground state. Assume that the nucleus is fixed at the origin.  
Hint: use Poisson's equation and recall that the ground state wave function of the electron is  $\psi = (\pi a^3)^{-1/2} \exp(-r/a)$ ,  $a = \hbar^2/me^2$ .

## Problem 2

Let  $a$  and  $a^\dagger$  be canonical annihilation and creation operators obeying  $[a, a^\dagger] = 1$ .

a) Find the most general linear transformation  $b = \alpha a + \beta a^\dagger$  such that  $b$  and  $b^\dagger$  are also canonical annihilation and creation operators.

b) Using the result of section a), find the ground state energy of the following Hamiltonian

$$H = A(\sqrt{3}aa + \sqrt{3}a^\dagger a^\dagger + 4a^\dagger a), \quad A = \text{constant.}$$

## Problem 3

Find the bound-state spectrum in the potential

$$U = -A/r + B/r^2,$$

where  $A$  and  $B$  are positive constants.

## Problem 4

An electron is subject to a uniform, time-independent magnetic field  $\vec{B} = (0, 0, B)$ . At  $t = 0$  the electron is in the  $+1/2$  eigenstate of  $S_x$ . Find the expectation value of  $S_x$  as a function of time.

## Problem 5

a) The quadrupole interaction of a spin-1 nucleus subject to an external inhomogeneous electric field can be written as

$$H = A(3S_z^2 - \vec{S}^2) + B(S_+^2 + S_-^2), \quad A, B = \text{constants.}$$

Write all the matrix elements  $\langle m' | H | m \rangle$  ( $m = 0, \pm 1$ ). What would the matrix elements of  $H$  be for a spin-1/2 nucleus?

b) Find the eigenvalues of the energy for the spin-1 nucleus.