New York University

Physics Department

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

QUANTUM MECHANICS II

Fall, 2001

READ INSTRUCTIONS CAREFULLY

1. ANSWER ALL PROBLEMS.
2. All problems have the same point value. When a problem is divided into parts, their relative weights are given as percentages in parentheses. If not otherwise indicated, all parts are equally weighted.
3. Write your own identification number on your answer booklets.
4. Show ALL your work.

Problem 1

An atom is initially in a state of angular momentum $L = 1$. It decays by dipole radiation into an intermediate state of angular momentum $L = 0$, which, in turns, rapidly decays into a ground state of angular momentum $L = 1$. The initial state has polarization $m = +1$, the final state has $m = -1$. Using *scccc c crccr* perturbation theory, find the probability $P(\phi)$ that the momentum vectors of the two emitted photons form an angle ϕ.

Problem 2

a

Let N electrons in a spherically symmetric potential have the same *aradaa* angular momentum l, and identical principal quantum number n. Using the Pauli principle, find the maximum value of N for a given l and n.

b

How many states does this system of N electrons, all with the same l and n, form? Here N need not be maximized. (Hint: use again Pauli's principle).

c

Suppose that our system is perturbed by a small spin-orbit term

$$H_{SO} = \sum_{i=1}^{N} A\vec{s}_i \cdot \vec{l}_i, \qquad A > 0.$$

Find the value of the total angular momentum J in the lowest-energy state with total orbital angular momentum L and total spin S.

d

Using Hund's rule and the results in parts a)-c) of this problem, find spin, orbital angular momentum and total angular momentum (S, L, J) for the ground state of the oxygen atom.

Problem 3

Using first-order time-dependent perturbation theory, find the transition rate for the ejection of an electron from a hydrogen atom in its ground state when perturbed by the potential

$$V = \mathrm{Re}\, A \exp i(kz - \omega t), \quad A = \text{constant}.$$

Assume that the nucleus of the atom is fixed at the origin, and that the final state is a plane wave.

Useful Formula

Ground-state wave function of the hydrogen atom:

$$\psi = (\pi a^3)^{-1/2} e^{-r/a}, \quad a = \frac{\hbar^2}{me^2}$$

Problem 4

a

Find the charge density for Z electrons in a spherically-symmetric potential $V(r)$ in the semiclassical approximation.

b

Using a), set up a self-consistent equation for the electrostatic potential of an atom in the semiclassical approximation (the Thomas-Fermi equation).

c

Using the Thomas-Fermi equation, find how the electrostatic energy of the atom depends on the atomic number Z.