New York University

Department of Physics

PRELIMINARY EXAMINATION FOR THE PH.D. DEGREE

Fall, 2001

Statistical Physics

READ INSTRUCTIONS CAREFULLY

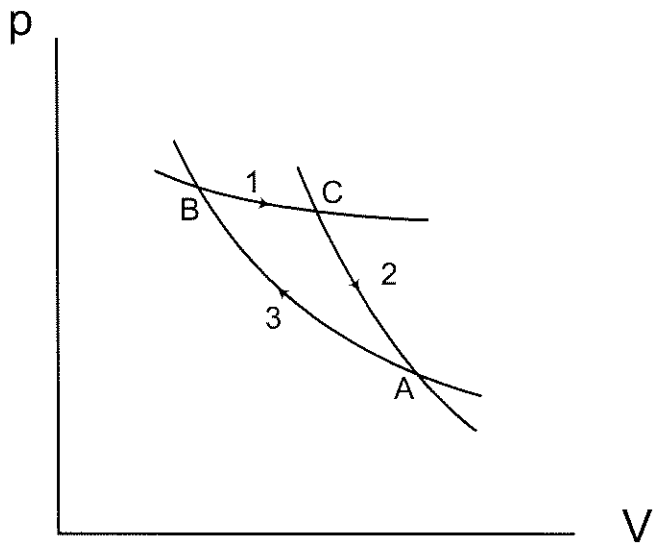
1. Answer all problems.
2. If not otherwise indicated, all parts of a problem are equally weighted.
3. Write your own identification number on your answer booklets.
4. Show all your work.

Problem 1

(10 points) It is found that the equation of state for a solid body is

$$V = a(T)p + b(T)p^2,$$

and this works well in the interval $p_1 \leq p \leq p_2$. Here $a(T)$ and $b(T)$ are some known functions of T. What is the entropy change that occurs when the body is compressed isothermally from p_1 to p_2 ?

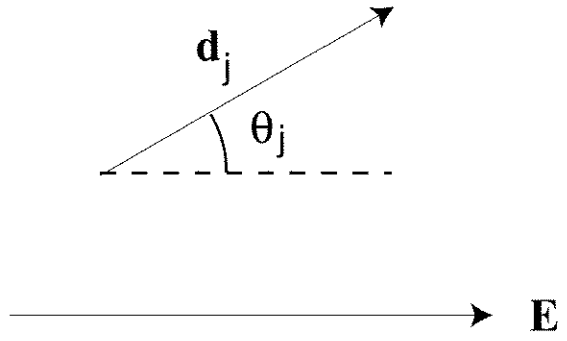


Problem 2

(10 points) The states of a gas are represented by points in the pV-plane. The slopes of isotherms through any point are smaller in magnitude than the slopes of the adiabats through the same point. Prove that adiabats cannot cross one another.(Hint : Consider the cycle shown in the figure.)

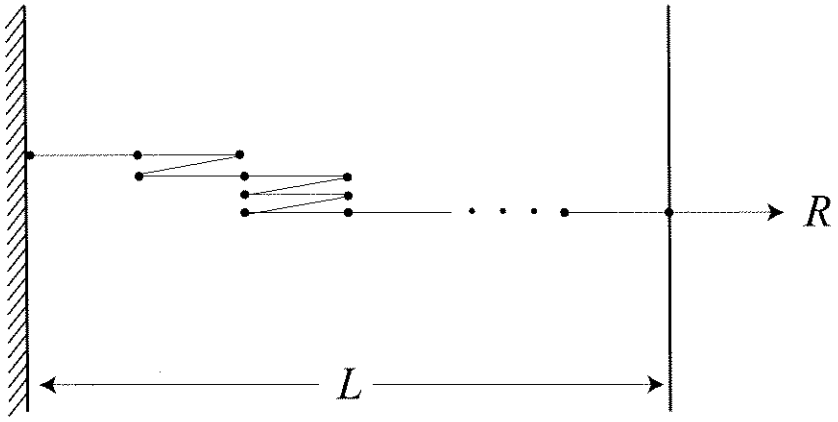
Problem 3

(20 points) Charged ions have spin $\frac{3}{2}$ and are constrained to move only along the z axis. A constant magnetic field B is applied along thez-direction. Find:
(a) the statistical sum(partition function) of the gas of ions;
(b) the mean energy $\langle E \rangle$, free energy F, and entropy S of the gas. Can we use the expression for entropy in the limit $T \to 0$? Explain your answer.



Problem 4

(30 points) A molecule of HCl can be considered as an electric dipole $\mathbf{d}$ arbitrarily oriented in space. The gas of N molecules is in a thermostat with temperature T. A constant electric field $\mathbf{E}$ is applied to the system along the z-direction (see figure). One may assume that the molecules do not interact among themselves. Find:
(a) the statistical sum (partition function) of the system.
(b) the polarization $\mathbf{P}$ of the system as the mean dipole moment $\langle \mathbf{D} \rangle$ of the full system.
(c) the fluctuations of the full dipole moment D_z along the field direction for high and low temperatures.



Problem 5

(30 points) A one-dimensional chain consists of $N >> 1$ equal elements of length a that can be oriented in either of two possible directions (see figure) and can reverse direction without change of energy. The chain is in a thermostat with temperature T and, due to thermal fluctuations one needs to apply a strain force R in order to keep the chain in equilibrium with a constant full length $L >> a$. Find:

(a) the total number of states of the chain for a given length L, and the entropy $S(L)$.

(b) the strain force $R(L, T)$. Show that the force is proportional to the length of the chain for $L << Na$.

Hint : The force R and length L are generalized force and generalized displacement, similar to p and V.