Minimum action paths for spin-torque assisted thermally induced magnetization reversal

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We calculate the most probable reaction paths for thermally induced magnetization reversal of a nanomagnet under the influence of spin transfer torque. The presence of the spin transfer torque implies that the standard reaction rate theory of Kramers cannot be used since the dynamics no longer shows detailed balance and so the magnetization reversals are nonequilibrium transitions. Thin film nanomagnets with a biaxial anisotropy, a shape anisotropy that leads to in-plane magnetization with a preferred axis in the plane, are considered. The reaction pathways and rates are computed using geometrical Minimum Action Method. Our results indicate that the transition state has an out-of-plane magnetization component, in contrast to the case without an applied spin transfer torque. © 2011 American Institute of Physics. [doi:10.1063/1.3565021]

There is considerable interest in controlling the magnetization in thin film magnetic elements using spin-polarized currents. The interaction between a spin current and background magnetization, known as spin transfer torque (STT),1–4 holds promise as a means of information storage and retrieval without the use of moving parts. A critical issue in designing STT devices is thermal stability: thermal fluctuations can cause spontaneous reversal of a desired magnetization state. What makes the analysis of these thermally induced reversals challenging is that the magnetization dynamics no longer shows detailed balance in the presence of STT, which implies that the reversals are nonequilibrium transition events and that the standard reaction rate theory of Kramers5 cannot be used to describe them. Early work on these reversals was carried out by Li and Zhang6 as well as Apalkov and Visscher.7 Here we calculate relaxation rates of STT systems using the geometrical Minimum Action Method (gMAM)8,9 to obtain the most probable transition paths and transition states of a single domain nanomagnet. Our results are consistent with the numerical simulations of Li and Zhang,6 and suggest the possibility of using the gMAM technique to study more complex problems involving nonequilibrium transitions.

We begin by distinguishing in which way nonequilibrium transitions differ from equilibrium ones. When the zero-noise dynamics is derivable from a potential $V$ and shows detailed balance, the thermally induced transitions occur at equilibrium and the Kramers theory predicts that the transition paths are time-reversed deterministic trajectories and that the transition rates $k$ (formally the inverse of the mean first passage time) are given by Arrhenius formula: $k = k_0 \exp(-\Delta V/k_B T)$, where $T$ is the temperature and $k_B$ is Boltzmann’s constant. In this formula, the leading-order asymptotics are governed by $\Delta V$, the potential energy difference to the transition state. The prefactor (or subdominant term) $k_0$ depends on the curvature of the potential at the local minimum and at the saddle along the minimum energy path (MEP). Kramers formula is asymptotically correct in the limit $T \to 0$; in practical problems, it is usually sufficient that $k_B T/\Delta V$ be small. Numerical methods to calculate the equilibrium transition paths and estimate $\Delta V$ include the Nudged Elastic Band Method10 and the String Method.11

In the presence of STT, the dynamics no longer satisfy the detailed balance condition, there is no longer a well-defined potential, and the magnetization reversals are nonequilibrium transitions. In particular, the transition paths are no longer time-reversed trajectories and the transition rates cannot be derived from Kramers theory. The appropriate formalism to calculate the transition paths and rates for these nongradient systems is the theory of large deviations.12 This theory gives an action whose minimizers are the most probable reaction pathways and expresses the reaction rate in terms of an Arrhenius-type formula in which the energy barrier is replaced by the minimum of the action. Large deviation theory has previously been used in field-driven magnetization reversal of macrospins13 and micromagnetics.14 It is also implicit in the work of Serpico et al.15 The gMAM (Refs. 8 and 9) is based on large deviation theory: it can be used to minimize the action functional and obtain the most probable transition paths and transition states of a single domain nanomagnet.

Here we focus on magnetic systems of the type studied by Li and Zhang,6 i.e., thin film nanomagnets of volume $v$, saturation magnetization $M_s$, and an in-plane anisotropy field $H_K$ directed along the $\hat{x}$ axis (the film is taken to lie in the $xy$ plane). We wish to find the most probable path $\mathbf{m}(t)$ connecting two magnetization states $\mathbf{m}_i$ and $\mathbf{m}_f$, which are dynamic attractors of the system in the absence of noise. If a spin-polarized current flows in the direction perpendicular to the film plane, then the dynamics are governed by the stochastic

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Landau–Lifshitz–Gilbert equation, which in the Stratonovich interpretation reads

\[ \dot{\mathbf{m}} = -\gamma' \mathbf{m} \times \mathbf{H}_{\text{eff}} + \gamma' \mathbf{m} \times [\mathbf{m} \times (\alpha J - z \mathbf{H}_{\text{eff}})] , \tag{1} \]

Here \( \gamma' = \gamma/(1 + x^2) \), \( \gamma \) is the gyromagnetic ratio, \( x \) is the damping constant, and \( \mathbf{P} \) is the current polarization vector oriented along \( m_x = +1 \). The strength of the spin-torque effect is determined by \( \alpha J \). The effective field contains deterministic and stochastic components

\[ \mathbf{H}_{\text{eff}} = -\nabla_m E(\mathbf{m})/\mu_0 M_s + \sqrt{\epsilon} \mathbf{W} , \tag{2} \]

where \( \mathbf{W} \) is a Gaussian white noise process, \( \epsilon = 2axk_B T/\mu_0 M_s \gamma \) measures the noise amplitude, and \( E \) is the micromagnetic energy density which, in the presence of an external field \( \mathbf{H}_{\text{ext}} \), reads

\[ E(\mathbf{m}) = -\mu_0 M_s H_{\text{ext}} \cdot \mathbf{m} - \frac{\mu_0 H_k M_s}{2} m_x^2 + \frac{\mu_0^2 M_s^2}{2} m_z^2. \tag{3} \]

The spin-torque interaction term in Eq. (1) breaks the detailed balance property of the dynamics. Equation (1) can be written as the following stochastic differential equation:

\[ \dot{\mathbf{m}} = \mathbf{b} + \sqrt{\sigma(\mathbf{m})} \mathbf{W} , \tag{4} \]

where

\[ \mathbf{b} = -K_A \left( \gamma' \nabla_m E/\mu_0 M_s \right) - \gamma' K_S \left( \gamma' \nabla_m E/\mu_0 M_s - \alpha J/P \right) . \tag{5} \]

is the deterministic drift vector and

\[ \sigma(\mathbf{m}) = -\gamma' (K_A + \gamma K_S) \tag{6} \]

is the diffusion matrix. Here \( K_S \) and \( K_A \) are symmetric and antisymmetric matrices with components

\[ K_{\text{S}_{\rho \sigma}} = (\delta_{\rho \sigma} - m_\rho m_\sigma), \quad K_{\text{A}_{\rho \sigma}} = \epsilon_{\rho \sigma \rho'} m_{\rho'} . \tag{7} \]

The presence of noise allows the system to deviate from the deterministic flow lines given by \( \mathbf{m} = \mathbf{b} = 0 \). From Eq. (4), the system’s response to an instantaneous thermal fluctuation is \( \mathbf{W} = e^{-1/2 \sigma^{-1}}(\mathbf{m} - \mathbf{b}) \). Since the probability of these fluctuations on the interval between times \( t_0 \) and \( t_f \) is proportional to \( \exp(-1/2 \sigma^{-1} \mathbf{W}^2 dt) \), a given trajectory occurs with probability proportional to \( \exp(-\epsilon^{-1} S) \) where the action \( S \) is defined as

\[ S = \frac{1}{2} \int_{t_0}^{t_f} ds \sigma^{-1}(\mathbf{m} - \mathbf{b})^2 dt. \tag{8} \]

Hence, the most probable path is the trajectory that minimizes this action. This trajectory is obtained with the gMAM algorithm. Furthermore, the value of the minimum action between \( \mathbf{m} \) and \( \mathbf{m_f} \) defines a pseudopotential \( V \), which provides information of the system statistical properties. In particular, the transition rates between states follow an Arrhenius-type formula \( \lambda_{1,2} \sim \exp(-1/2 V(\mathbf{m}_1, \mathbf{m}_2)) \).

Figure 1 shows typical trajectories obtained using the gMAM algorithm. For comparison purposes the deterministic trajectories are also shown. The basin of attraction of the \( m_x = +1 \) contracts with increasing current.

The minimum action paths present kinks at the transition points \( (\mathbf{b} = 0) \). This is explained as follows: the trajectory from the transition points to the attractive critical points (downstream) follows the deterministic drift lines. The segment running from the attractive critical point to the transition point (upstream) needs to overcome the drift field, but the action is a maximum if the trajectory is antiparallel to the drift fields. The action minimization can be achieved by a trajectory that is not entirely antiparallel to the drift lines. The sharp change of direction at the transition point occurs because the trajectory switches from upstream to downstream.

For finite currents the critical points of Eq. (5) no longer coincide with the saddle points of the energy surface. The effect of currents on the minimum action paths is shown in Fig. 1. It is evident that the transition state now has an out-of-plane magnetization component. This arises because the coefficient of \( K_S \) and \( K_A \) are not simultaneously zero. A positive current favors \( m_x = -1 \) and shifts the critical point toward \( m_x = +1 \). With this in-plane displacement of the magnetization, the precessional term is no longer zero and needs

![FIG. 1. (Color online) Deterministic flow lines (black) and minimum action paths for the magnetization. The forward (red and above the plane, from \( m_x = -1 \) to \( m_x = +1 \)) and backward (green and below the plane, from \( m_x = -1 \) to \( m_x = +1 \)) minimum action paths are not only different, but they cannot be obtained from the deterministic trajectories by time reversal under detailed balance except when \( \alpha J = 0 \). Indeed, if \( \alpha J \) is nonzero, the STT term is present and breaks the detailed balance condition: in these cases, the transitions occur out of equilibrium. The shaded area is the basin of attraction of point \( m_x = 1 \); unshaded, of \( m_x = -1 \). For finite values of \( \alpha J \), the transition point moves out of the plane. The external field is \( H_z = 0.01 \) T. (Here we set \( \mu_0 M_s = 0.12 \) T, \( x = 0.5 \), and \( \mu_0 H_k = 0.05 \) T for easier visualization.)](https://link.to/image)
to be counterbalanced by an upward shift. As the magnetization moves upward, the shape anisotropy field becomes non-zero and acts downward on the magnetization. The net effect is that the current produces a leftward and upward shift of the transition magnetization. The critical points now have an out-of-plane magnetization component. This simple result may have important consequences on current understanding of nonuniform magnetization reversal of thin films: while it is known that stationary points in the energy surface of thin films occurs for in-plane magnetizations, the effect described here shows that when a spin torque is present, thermal transitions may have an out-of-plane magnetization component.

In what follows we focus on transitions from \( m_0 = -1 \) to \( m_f = +1 \). To obtain the dependence of the action on the current strength we evaluated the minimum action path in a two-step procedure. First, we use gMAM to find an approximate action path that allows us to visually identify the location of the transition point. We discard the downstream section of the path and recalculate the action using gMAM only in the upstream direction. That value is our estimate for the minimum action.

We now present results of the action’s dependence on field and current magnitude in Fig. 2. The action is calculated for trajectories from \( m_0 = -1 \), to \( m_f = +1 \). In a magnet with \( \mu_0 M_s = 1.2 \text{ T} \), \( \alpha = 0.05 \), \( \mu_0 H_K = 0.05 \text{ T} \), for which \( E_0 = 59.02 \text{ kBT} \). The action has positive values and approaches zero as the switching field is reached. Positive currents result in larger values for the action.

By calculating the rate of the transition and using it in an Arrhenius-type formula, Li and Zhang\(^6\) describe an effective activation energy, which in our geometry reads:

\[
E_b = E_0 (1 - H_s/H_K)^2 (1 + a_j/a_c), \tag{9}
\]

where \( E_0 \) is the activation energy at zero current and zero field, and \( a_c = x(H_s + H_K + M_s/2) \) is the critical spin current. This effective energy can be compared directly to the minimum of the action calculated using gMAM. While our results do present a quadratic dependence on the external field, as can be seen in Fig. 2, we have found a different dependence of the action on the current magnitude \( a_j \). Our results cover regimes where the energy barrier is high with respect to thermal noise, while the numerical approach of Li and Zhang requires applied fields close to the switching fields. With gMAM we are able to explore a wider region of the parameter space.

Summarizing, we have calculated minimum action paths and corresponding actions for magnetic particle under the influence of spin torque. Our results indicate that the transition state has an out-of-plane magnetization component, while previously they have been considered to be in-plane magnetizations. The Minimum Action Method promises to be a useful tool for the study of a large variety of magnetic systems.

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