# Modern Physics I
# Solutions to Homework 2 Handout

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Bernstein, Fishbane, Gasiorowicz: Chapter II, problems 13-25.

13. The square of the interval is an invariant:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \frac{(x - vt)^2}{1 - \frac{v^2}{c^2}} + y^2 + z^2 - c^2 \frac{\left(t - \frac{v}{c^2}x\right)^2}{1 - \frac{v^2}{c^2}} =$$

$$= \frac{x^2 - 2xvt + v^2t^2 - c^2t^2 + 2tvx - \frac{v^2}{c^2}x}{1 - \frac{v^2}{c^2}} + y^2 + z^2 = x^2 + y^2 + z^2 - c^2t^2. \quad (1)$$

14. We have a system of two linear equations with respect to $x$ and $t$:

$$\begin{cases} x' = \gamma(x - vt), \\ t' = \gamma\left(t - \frac{v}{c^2}x\right) \end{cases} \quad (2)$$

The solutions can be obtained, for example, by Cramer formulas

$$x = \frac{\begin{vmatrix} x' & -\gamma v \\ t' & \gamma \end{vmatrix}}{\begin{vmatrix} \gamma & -\gamma v \\ -\gamma\frac{v}{c^2} & \gamma \end{vmatrix}} = \frac{\gamma(x' + vt')}{\gamma^2 - \gamma^2\frac{v^2}{c^2}} = \gamma(x' + vt'),$$

$$t = \frac{\begin{vmatrix} \gamma & x' \\ -\gamma\frac{v}{c^2} & t' \end{vmatrix}}{\begin{vmatrix} \gamma & -\gamma v \\ -\gamma\frac{v}{c^2} & \gamma \end{vmatrix}} = \frac{\gamma\left(t' + \frac{v}{c^2}x'\right)}{\gamma^2 - \gamma^2\frac{v^2}{c^2}} = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad (3)$$

15. (a) The speed of the rockets relative to each other is

$$v = \frac{0.8 + 0.6}{1 + 0.8 \times 0.6} c = 0.946c. \tag{4}$$

(b) The contracted lengths of the rockets, that is their lengths as seen from the Earth are

$$l'_A = l_A \sqrt{1 - \beta_A^2} = 300\,m, \tag{5}$$

$$l'_B = l_B \sqrt{1 - \beta_B^2} = 800\,m. \tag{6}$$

Their tails will be together after they travel a length $l'_A + l'_B$ at a speed $0.8c + 0.6c$, that is at time

$$\frac{1100\,m}{1.4 \times 3 \times 10^8\,m/s} = 2.62\,\mu s \tag{7}$$

after their noses were together.

16. This time is the length of rocket $B$ in $A$'s frame divided by the speed of rocket $B$ in $A$'s frame:

$$\frac{1000\,m\,\sqrt{1 - 0.946^2}}{0.946 \times 3 \times 10^8\,m/s} = 1.14\,\mu s. \tag{8}$$

17. This time is the length of $A$ in $B$'s frame plus the proper length of $B$ all divided by the speed of $A$ in $B$'s frame:

$$\frac{500\,m\,\sqrt{1 - 0.946^2} + 1000\,m}{0.946 \times 3 \times 10^8\,m/s} = 4.1\,\mu s. \tag{9}$$

18. The pole's front end will arrive to the front door of the shed before the back end. The rear door will shut later, only when the back end of the pole reaches it.

19. The light you observe when moving towards the emitter will have wavelength

$$\lambda' = \sqrt{\frac{1 - \beta}{1 + \beta}} \lambda \sqrt{1 + \frac{1}{2} 1 - \frac{1}{2}} 580\,nm = 335\,nm, \tag{10}$$

not visible light but ultraviolet. The answer is the same if the emitter moves toward you.

20. The frequency will be red shifted:

$$f' = \sqrt{\frac{1-\beta'}{1+\beta'}} f = \sqrt{\frac{1-0.8}{1+0.8}} 1.2 \times 10^{15}\, Hz = 4 \times 10^{14}\, Hz. \tag{11}$$

21. The frequency will be red shifted further

$$f'' = \sqrt{\frac{1-\beta''}{1+\beta''}} f' = \sqrt{\frac{1-0.6}{1+0.6}} 4 \times 10^{14}\, Hz = 2 \times 10^{14}\, Hz. \tag{12}$$

22. The Doppler shift formula

$$f'' = \sqrt{\frac{1-\beta}{1+\beta}} f \tag{13}$$

gives

$$\beta = \frac{1-\left(\frac{f''}{f}\right)^2}{1+\left(\frac{f''}{f}\right)^2} = \frac{35}{37} \approx 0.946. \tag{14}$$

One can get it also by

$$\frac{0.8+0.6}{1+0.8\times 0.6} = 0.946 \tag{15}$$

23. (a)

$$f' = \sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} f. \tag{16}$$

(b)

$$f'' = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} f'. \tag{17}$$

(c) The relationship between $f''$ and $f$ must be of the form

$$f'' = \sqrt{\frac{1-\frac{w}{c}}{1+\frac{w}{c}}} f. \tag{18}$$

with some speed $w$. Using (a) and (b) we can write

$$f'' = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}f. \tag{19}$$

Matching the coefficients, we get

$$\sqrt{\frac{1-\frac{w}{c}}{1+\frac{w}{c}}} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}. \tag{20}$$

Solving for $w$,

$$1-\frac{w}{c} = \frac{1-\frac{v}{c}}{1+\frac{v}{c}}\frac{1-\frac{u}{c}}{1+\frac{u}{c}}\left(1+\frac{w}{c}\right), \tag{21}$$

$$c-w = \frac{c^2-vc-uc+uv}{c^2+vc+uc+uv}(c+w), \tag{22}$$

$$w = \frac{u+v}{1+\frac{uv}{c^2}}, \tag{23}$$

which is the addition of velocities formula.