

Stability of 2π domain walls in ferromagnetic nanorings

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The stability of 2π domain walls in ferromagnetic nanorings is investigated via the calculation of the energy barrier that separates a 2π domain wall from the vortex state of a ferromagnetic nanoring. Trapped domains are stable when they exist between certain types of transverse domain walls, i.e., walls in which the edge defects on the same side of the magnetic strip have equal sign and thus repel. Here the energy barriers between these configurations and the counterclockwise magnetization state are obtained using the string method. Due to the geometry of the rings, two types of 2π walls must be distinguished that differ by their overall topological index and exchange energy. The minimum energy path corresponds to the expulsion of a vortex. The energy barrier for annihilation of a 2π wall is compared to the activation energy for transitions between the two ring vortex states.

I. INTRODUCTION

Recent observations¹⁻³ in thin ferromagnetic stripes show magnetization configurations in which the magnetization makes a full 2π turn in a localized region of the stripe, while the rest of the stripe is magnetized parallel to the edges of the stripe. These structures are stable against small applied external magnetic fields which makes them potentially useful for information storage devices⁴. We have previously observed the same phenomenon in thin ferromagnetic annuli that results in the existence of a hierarchy of metastable states⁵. The location of a 2π domain wall in rings has recently been proposed as a mean to encode information⁴.

Reliable control of domain wall structures is crucial in the successful design of magnetic nanodevices^{6,7}. Here we explore the stability of 2π domain walls in ferromagnetic nanorings using the string method⁸⁻¹⁰. We find the energy barrier ΔE separating two metastable configurations which determines the escape rate from an stable state as being proportional to $e^{-\beta\Delta E}$ where β is the inverse thermal energy¹¹. The string method also gives the most probable escape path and transition state.

We distinguish two types of domain walls by their winding number in the global (local) coordinate system ω (Ω). We compare the energy barriers that separates each type of wall from the ground state to the activation energy that separates the two vortex configurations (instanton saddle^{5,12}). A current flowing along the axis of the ring produces a circumferential field. The application of the circumferential field has two effects. First, the degeneracy of the two vortex states is lifted. Second, the field determines the width of the domain wall; in its absence the 2π walls could dissociate into two independent transverse walls at arbitrary distances from each other.

II. METHOD

Following previous work^{5,12}, we study a permalloy ring (exchange constant $A = 1.3 \times 10^{-11}$ J/m) with the following dimensions and material properties: outer radius

$R_2 = 220$ nm; inner radius $R_1 = 180$ nm, thickness $t = 2$ nm, magnetization saturation $M_s = 8 \times 10^5$ A/m, and exchange length given by $l_{\text{ex}} = \sqrt{\frac{2A}{\mu_0 M_s^2}} = 5.6$ nm. A current flowing along the axis of the ring exerts a field $\mathbf{H}(\mathbf{r}) = (hH_c(R_1 + R_2)/2r)\hat{\theta}$ A/m on the ring. Here h represents a fraction of the characteristic field $\mu_0 H_c = 73.9$ mT (for $H > H_c$, the clockwise vortex state is no longer stable).

We consider the overdamped case so that the escape trajectory follows the negative gradient of the energy. This is done by integrating only the damping term of the Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{M}}{dt} = -\frac{|\gamma|\alpha}{M_s}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}). \quad (1)$$

Here α is the damping coefficient, γ is the gyromagnetic constant, and $\mathbf{H}_{\text{eff}} = -\nabla_{\mathbf{M}}E/\mu_0$ is the effective magnetic field. The total micromagnetic energy E is the sum of the exchange E_{ex} , Zeeman E_Z and magnetostatic terms E_{mag} .

The string method is used to calculate the minimum energy path $\xi(\alpha)$, ($\xi : \alpha \in [0, 1] \rightarrow \mathbf{M}(\mathbf{r})$) between two stable states ($\xi(0) = \mathbf{M}_{\mathbf{A}}, \xi(1) = \mathbf{M}_{\mathbf{B}}$) that passes through the minimum energy barrier that separates them. In practice, the path is discretized in $N+1$ points between $\mathbf{M}_{\mathbf{A}}$ and $\mathbf{M}_{\mathbf{B}}$ denoted as $\mathbf{M}_i(\mathbf{r}, t)$ with $i = 0, \dots, N$. Each magnetization evolves using the publicly available micromagnetic code OOMMF¹³ until the time reaches some interval Δt which we have selected to be 10 ps. This gives a sequence of configurations:

$$\mathbf{M}'_i(\mathbf{r}, t) = \mathbf{M}_i(\mathbf{r}, t) + \int_t^{t+\Delta t} \frac{d\mathbf{M}_i(\mathbf{r}, t')}{dt'} dt' \quad (2)$$

Once all the $\mathbf{M}'_i(\mathbf{r}, t)$ have been obtained, the string method requires a reparametrization step. First the complete arc s_N length of the trajectory is calculated by

$$s_0 = 0, s_i = s_{i-1} + |\mathbf{M}'_i(\mathbf{r}, t) - \mathbf{M}'_{i-1}(\mathbf{r}, t)|. \quad (3)$$

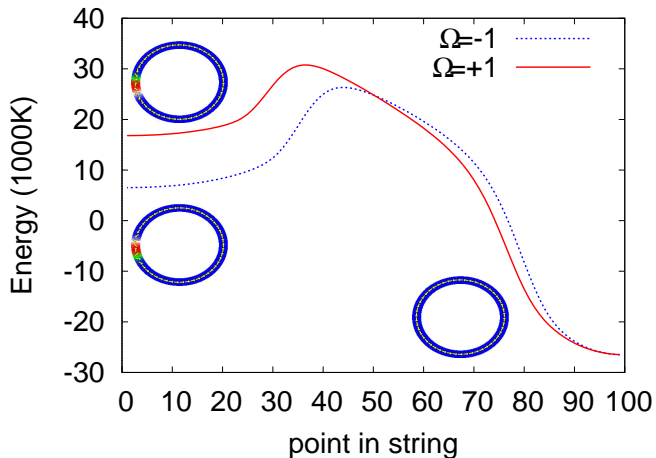


Figure 1: Energy barrier for annihilation of 2π domain wall.

The arc lengths are renormalized using $\alpha'_i = s_i/s_N$. Finally we do a simple linear interpolation for all i along the trajectory so that

$$\mathbf{M}_i(\mathbf{r}, t + \Delta t) = \mathbf{M}'_j(\mathbf{r}, t) + \frac{\mathbf{M}'_{j+1}(\mathbf{r}, t) - \mathbf{M}'_j(\mathbf{r}, t)}{\alpha'_{j+1} - \alpha'_j} \left(\frac{i}{N} - \alpha'_j \right) \quad (4)$$

where $j(i)$ is the index of the string where $\alpha'_{j+1} \geq i/N \geq \alpha'_j$. During each step of the program we observe the magnetic energy $E_i(t) = E_{\text{ex}}(\mathbf{M}_i(\mathbf{r}, t)) + E_Z(\mathbf{M}_i(\mathbf{r}, t)) + E_{\text{mag}}(\mathbf{M}_i(\mathbf{r}, t))$ as indicator of how far from convergence the current step is. The iteration process is stopped when there is no visible change in the function $E_i(t)$.

III. ANNIHILATION OF 2π DOMAIN WALL.

We now present the results of the string method to find the minimum energy path for destruction of a 2π wall for the two types of 2π domain wall described. Fig. 1 and its companion Fig. 2 present the string energies and configurations after relaxation of the string. The $\Omega = +1$, 2π wall decays into the counterclockwise configuration by the expulsion of a vortex from the inner hole. On the other hand, the $\Omega = -1$, 2π wall decays into the counterclockwise configuration by the expulsion of an antivortex. This observation shows a remarkable correspondence between a topological defect crossing the stripe and the signature of the 2π walls being annihilated.

For comparison purposes we provide magnitudes of the energy landscape of ferromagnetic nanorings obtained with the string method. The lowest energy barrier between the counterclockwise and the clockwise vortex configurations passes through the instanton saddle (with $\Omega = 0$); the activation energy of this event is 22×10^3 K. This is consistent with our previous work on nanorings⁵. For a 2π wall with index $\Omega = 1$ the decay into the ground

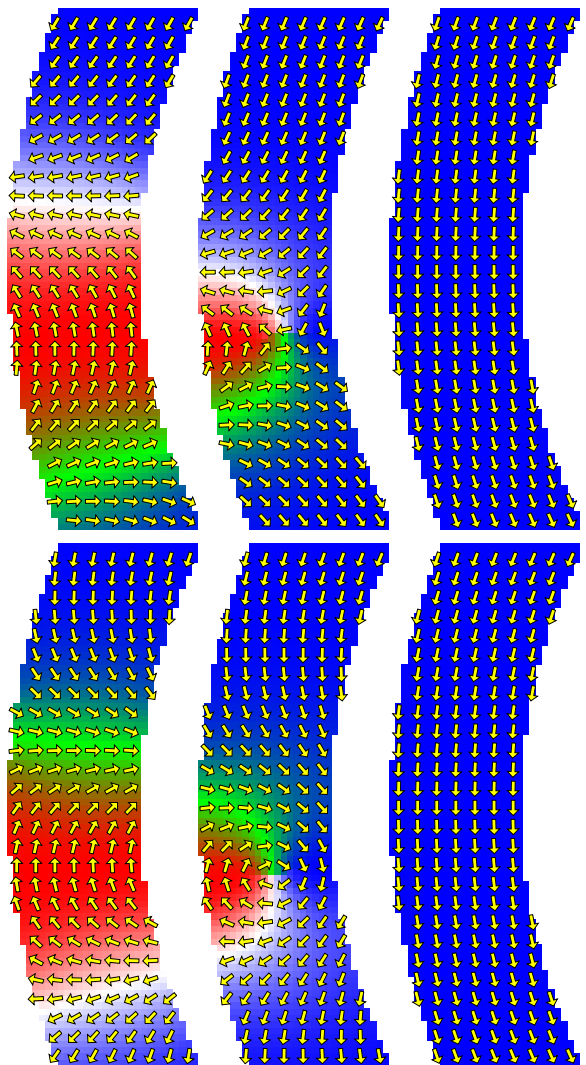


Figure 2: Minimum energy path for the annihilation of 2π domain walls in consideration. (Above) 2π wall with topological index $\Omega = -1$, (below) 2π wall with topological index $\Omega = +1$.

state has an energy barrier equal to 14×10^3 K. The annihilation a 2π wall with index $\Omega = -1$ has an energy barrier equal to 20×10^3 K. This shows that the annihilation of a 2π domain wall by the expulsion of a topological defect is comparable to the reversal between vortex states by a instanton fluctuation.

IV. DISCUSSION

For annuli with the dimensions being considered the magnetization is constrained to lie on the plane of the ring; for the stable states the magnetization can be considered to be independent of the radial coordinate. The exchange energy E_{ex} of a stable state is given by¹²:

$$E_{\text{ex}} = \frac{\mu_0 M_s^2 t_{\text{ex}}^2}{2} \ln \left(\frac{R_2}{R_1} \right) \left[2\pi(1 - 2\Omega) + \int_0^{2\pi} \left(\frac{\partial \phi}{\partial \theta} \right)^2 d\theta \right] \quad (5)$$

where $\phi(\theta)$ is the angle that the magnetization makes with the tangent of the ring at a given angle θ ; Ω is a “winding number” of the magnetization with respect to the local coordinate system.

The difference in winding numbers ($\Delta\Omega$) of the 2π walls considered results in an exchange energy difference ΔE_Ω between these two states. Using Eq. (5) the difference can be shown to be approximately:

$$\Delta E_\Omega[\mathbf{M}(\mathbf{r})] = 2\pi\mu_0 M_s^2 t_{\text{ex}}^2 \ln \left(\frac{R_2}{R_1} \right) \Delta\Omega = 9,401 \text{ K}. \quad (6)$$

The total exchange energy difference between these 2π walls ($\Omega = \pm 1$) obtained from the micromagnetic simulation results is 9,827K. Here we have ignored a contribution to this difference of terms of the form $\int \left(\frac{\partial \phi}{\partial \theta} \right)^2 d\theta$ since, in equilibrium, they combine with the demagnetization and Zeeman energy terms into an energy minimum. This shows a very good agreement between an estimate obtained from the 1D model and the full numerical simulation. The exchange energy term is the biggest contributor to the difference between the total energies of the two domain walls: the numerical values from the demagnetization and Zeeman energy are 10 times smaller. The main point is that most of the energy difference between these two types of 2π domain wall is the result of their respective topological windings.

We now consider the question of how to produce these two types of 2π walls. We use the information contained in their global topological number ω and compare it to other known states of nanorings. In particular, the well-known ‘onion’ state has $\omega = 0$. Since the onion is the remanent magnetization after saturation on an inplane uniform field, one can produce the $\Omega = -1$ wall by applying a strong field in-plane, and applying the circumferential field so that domain sizes change accordingly. The two walls will approach and form a $2\pi, \Omega = -1$ wall. Changing the direction of either the in-plane field or the circumferential field will only change the final position of the 2π wall, not its topological index.

The $2\pi, \omega = 2$ domain wall cannot be produced using only uniform and circumferential fields. However, we propose the following technique to produce that configuration in nanorings: apply a strong dipolar field ($\omega = 2$) in the interior of the ring. This could be produced by a small current loop with its axis coplanar to the structure, or by bringing a magnetic tip close to the device. If this dipolar field is strong enough, two transverse walls would be produced at opposite sides of the ring; the magnetization vector at the centers of the wall will point in the opposite direction of the overall magnetization of the ring. Activating the circumferential field as the dipole

strength is decreased will result in the desired configuration.

One final question is whether resistance measurements can distinguish the difference between the two types of 2π structures reported in this work. For instance, one could attempt to use Anisotropic Magnetoresistance experiments to read the overall winding number of the configuration. An estimate of this effect can be obtained by integrating $\mathbf{J} \cdot \mathbf{M}$ along a certain segment of the ring that spans the whole 2π domain wall. Since \mathbf{J} runs along $\hat{\theta}$ the AMR would be proportional to $\langle M_\theta / M_s \rangle$ which can be directly calculated from the 2π configuration. For two electrodes located at the top and bottom of the segment shown in Fig. 2 these values are 0.042 and 0.016 for the $\Omega = +1$ and $\Omega = -1$ walls respectively. It may be possible to apply a current to probe the winding number of the 2π domain wall as an extra splitting of the AMR signal.

V. CONCLUSION

We have presented results on the thermal annihilation of 2π domain walls. We differentiated between two types of 2π walls through their winding number in curved nanowires. We have observed a simple arithmetical relation between the topological index of the different configurations and the processes by which each structure decays into the ground state. The fact that the energy difference between the two states is dominated by the exchange energy allows to identify the states through their winding number. The transition path requires the motion of a singularity through the bulk: an antivortex destroys $\Omega = -1$ walls, and a vortex annihilates $\Omega = +1$ walls. Similar behavior might be expected to work for linear stripes. Topological defects are known to play a role in certain types of phase transitions; here we have detailed the mechanism by which they destroy 2π wall structures.

The two types of 2π domain wall correspond to distinct metastable states: the greatest contribution to the energy difference comes from the exchange energy difference. The energy can be directly associated with the topological signature of the magnetization configuration. The stability of these two states is comparable to the stability of the clockwise configuration. We suggest to use AMR telegraph noise measurements as a way to verify the presence of the two types of wall in magnetic nanowires. Multiple thermally activated reversal should generate both types of wall which could be recognized as two separate values of the AMR.

Further micromagnetic exploration of the 2π wall annihilation problem presented here can be done moving away from the overdamped regime by using a nonzero precessional term. The results presented here are a convenient limiting case for comparison purposes.

VI. ACKNOWLEDGMENTS

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