READ INSTRUCTIONS CAREFULLY

1. Answer all problems.
2. If not otherwise indicated, all parts of a problem are equally weighted.
3. Write your own identification number on your answer booklets.
4. Show all your work.

Problem 1

(10 points) It is found that the equation of state for a solid body is

\[ V = a(T)p + b(T)p^2, \]

and this works well in the interval \( p_1 \leq p \leq p_2 \). Here \( a(T) \) and \( b(T) \) are some known functions of \( T \). What is the entropy change that occurs when the body is compressed isothermally from \( p_1 \) to \( p_2 \)?
Problem 2

(10 points) The states of a gas are represented by points in the $pV$-plane. The slopes of isotherms through any point are smaller in magnitude than the slopes of the adiabats through the same point. Prove that adiabats cannot cross one another. (Hint: Consider the cycle shown in the figure.)

Problem 3

(20 points) Charged ions have spin $\frac{3}{2}$ and are constrained to move only along the $z$ axis. A constant magnetic field $B$ is applied along the $z$-direction. Find:
(a) the statistical sum (partition function) of the gas of ions;
(b) the mean energy $\langle E \rangle$, free energy $F$, and entropy $S$ of the gas. Can we use the expression for entropy in the limit $T \rightarrow 0$? Explain your answer.
Problem 4

(30 points) A molecule of HCl can be considered as an electric dipole $d$ arbitrarily oriented in space. The gas of $N$ molecules is in a thermostat with temperature $T$. A constant electric field $E$ is applied to the system along the $z$-direction (see figure). One may assume that the molecules do not interact among themselves. Find:

(a) the statistical sum (partition function) of the system.
(b) the polarization $P$ of the system as the mean dipole moment $\langle D \rangle$ of the full system.
(c) the fluctuations of the full dipole moment $D_z$ along the field direction for high and low temperatures.
Problem 5

(30 points) A one-dimensional chain consists of $N >> 1$ equal elements of length $a$ that can be oriented in either of two possible directions (see figure) and can reverse direction without change of energy. The chain is in a thermostat with temperature $T$ and, due to thermal fluctuations one needs to apply a strain force $R$ in order to keep the chain in equilibrium with a constant full length $L >> a$. Find:

(a) the total number of states of the chain for a given length $L$, and the entropy $S(L)$.

(b) the strain force $R(L, T)$. Show that the force is proportional to the length of the chain for $L << Na$.

Hint: The force $R$ and length $L$ are generalized force and generalized displacement, similar to $p$ and $V$. 